N-body theory of near-field heat transport

P. Ben-Abdallah ¹, S.A. Biehs ², K. Joulain ³
LCFIO, Palaiseau, France
Institut für Physik, Oldenburg, Germany
Institut P', Poitiers, France

“Eurotherm91, Poitiers August 2011”
The beginning of the story

Far-field

Near-field

Propagative modes

Evanescent modes

$S = \frac{\pi^2 k_B^4}{60 \hbar^3 c^2} (T_1^4 - T_2^4)$

Near field

$\sim 1/d^2$

Far field

$d \gg \lambda_T$

$d << \lambda_T = c h / (k_B T)$

Some theoretical predictions

A. Narayanaswamy and G. Chen, PRB, 77, 2008


A. W. Rodriguez and al., ArXiV, 2011


S.-A. Biehs et al., Optics Express 19, 2011.

K. Joulain et al. PRB, 81, 165117 (2010)
Some experimental confirmations

Shen et al. Nanoletters 2009

Ottens et al. PRL 2011

Hu et al. APL 2008

Kittel et al. PRL 2005

Rousseau et al. Nature Photonics 2009

sphere-plane geometry
(needs no angular alignment)

plane-plane geometry
Open questions
(in complex mesoscopic systems)

- How does the heat transport for a collection of individual objects in mutual interaction look like?
- What are the heat propagation regimes in disordered systems (role of localized and delocalized modes)?
- What are the percolation thresholds in nanocomposites structures?

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Needs a theory to describe N-body heat transfers
Outline

- Description of N-body system
- Landauer-like formulation of heat transport in complex systems
- Many body effects in N-body systems
- Conclusions
Description of N-body system

\[ p_j = p_j^{\text{fluc}} + p_j^{\text{ind}} \]

\[ \mathbf{E}_{ij} = \mu_0 \omega^2 G_0^{ij} p_j^{\text{fluc}} + \frac{\omega^2}{c^2} \sum_{k \neq i} G_0^{ik} \alpha_k \mathbf{E}_{kj} \]

N discrete objects

N dipoles

Coupled dipoles (Foldy-Lax, 1952)

\[ \mathbf{p}_i^{\text{ind}} = \varepsilon \alpha_i \sum_{j \neq i} \mathbf{E}_{ij} \]

induced dipole moment

\[ d_i \ll \lambda_{Tj} \]
Landauer-like formulation of heat transport

Power exchanged between two objects:

\[ P_{j \rightarrow i} \propto \int_{V_i} \text{Re}[j_i^* E_j] dV_i \]  

- Poynting theorem

\[ \langle p_{j,\alpha}^* p_{i,\beta} \rangle = 2 \varepsilon_0 \frac{c_0}{\omega} \text{Im}(\alpha_j) \Theta(\omega, T_j) \delta_{\alpha\beta} \delta_{ij} \]  

\[ P_{j \rightarrow i} = 3 \int_0^\infty \frac{d\omega}{2\pi} \Theta(\omega, T_j) T_{i,j}(\omega) \]  

with  
\[ T_{i,j}(\omega) = \frac{4}{3} \frac{\omega^4}{c^4} \text{Im}(\alpha_i) \text{Im}(\alpha_j) \text{Tr}[G_{ij}^\dagger G_{ij}] \leq 1 \]  

Transmission coefficient
Landauer-like formulation of heat transport

\[ T_j = T_i + \Delta T \]

Introducing the heat conductance between two objects:

\[ G_{ij} = \frac{\partial P_{j \rightarrow i}}{\partial T_j} \quad \text{(Linearization of exchanged power)} \]

\[ P_{j \rightarrow i} = 3 \left( \frac{\pi^2 k_B^2 T}{3h} \right) \mathcal{T}_{i,j} \Delta T \]

(Pendry, Math. Gen., 1983
Schwab, Nature 200)

\[ \mathcal{T}_{i,j} = \frac{3}{\pi^2} \int dx \frac{x^2 e^{-x}}{(e^x - 1)^2} \mathcal{T}_{i,j} \]

Mean transmission coefficient

3 channels for heat exchanges
Many body effects in N body systems

SiC particles

$T_1 = 300 \text{ K}$

$T_2 = 0 \text{ K}$

$T_3 = 0 \text{ K}$

$R = 100 \text{ nm}$

$pba et al. PRL sept 2011$

$\varphi_{12}(x_3, y_3)$

Heat transfer can be either magnified or inhibited by the third particle.
Many body effects in N body systems

\[ T_{i,j}^{(N)}(\omega) = \frac{4}{3} k_0^2 \text{Im} \alpha \text{tr}[G_N G_N^+] \]

More efficient coupling at longer separation distances with a third particle

\[ T_{i,j}^{(2)}(\omega_{SR}, l = 3R) = 0.12 \]

\[ T_{i,j}^{(3)}(\omega_{SR}, l = 3R) = 0.3 \]
Conclusions

We have:

- introduced a framework to investigate the NF heat transport in complex N-body systems.
- derived a Landauer-like formulation of heat transfer problem
  - Found the fundamental limits for the transfer
- Shown that the N-body interactions can magnify/inhibit heat transport

This physics could find broad applications in TPV energy conversion technologies and for thermal management